① Revise POSET

Note: if two elements are related they are called comparable otherwise non comparable

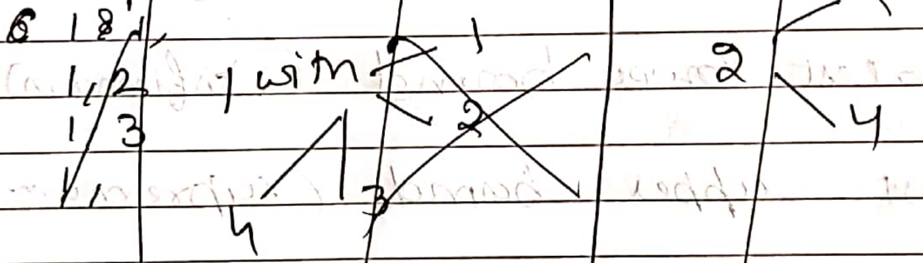
② Comparable elements & Non comparable elements.

Let $A = \{1, 2, 3, 4\}$
relation is divisibility
then

1 $R = \{(1,1), (1,2)\ (1,3), (1,4), (2,2), (2,4)\}$

~~Excluding the elements it self~~

we Comparable — Excluding the element



Non comparable (2,3) (3,4)

~~(2, 3)~~

either $a \leq b$ or $b \leq a$ Comparable

if neither $a \leq b$ nor $b \leq a$ then non-comparable

Acc. to example
2 cannot be compared with 3
or 2 and 3 are non-comparable
IIly 3 and 4 are non-comparable

Note:- If every pair is comparable then it is said to be linearly/ fully/ complete ordered.

Maximal & Minimal elements

Maximal : Let A be a POSET then $a \in A$ is said to be maximal element if there does not exist $c \in A$ s.t $a \leq c$

Minimal

s.t $a \geq c$

Note:- More than one minimal & Maximal elts can exist.

Lower Bound : Let B be a subset of A where A is POSET then $x \in A$ is lower bd. of B if $x \leq y$ $\forall y \in B$

Upper bound : s.t $x \geq y$ $\forall y \in B$

Greatest lower bound (Infimum) :

least upper bound (Supremum

Let $A = \{1, 2, 3, 4, 5, 6\}$ with relation $\leq$

$B \subseteq A$ s.t $B = \{2, 4, 5\}$

then

upper Bound of $B = \{5, 6\}$

least upper bound $= 5$

lower Bound of $B = \{1, 2\}$

Greatest lower bound $= 2$

Hasse Diagram (Ordering Diagram)

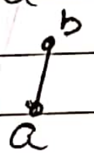
It is the graphical picture of POSET.

Since POSET is reflexive so there is no need to draw self loop on every element in hasse diagram

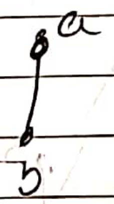
Since POSET is transitive
i.e aRb, $bRc \Rightarrow aRc$

No need to draw edge between a & c.

if aRb
then in diagram
b will exist above a.



Similarly bRa



[Arrow may be drawn or may omitted]

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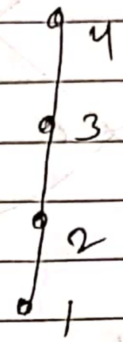
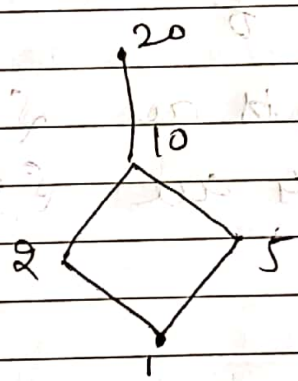
Ex A = {1, 2, 3, 4} ≤ then

Hasse diagram is



Ex-2 A = {1, 2, 5, 10, 20} divisibility



Note :- 1 cannot divisible by any no.
so it is base

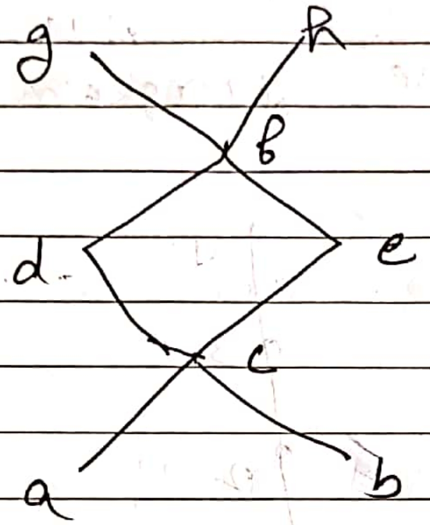
2 and 5 can be divided by only ~~$\emptyset$~~ 1
so these are parallel and just adjacent to 5

10 can be divisible by both
so it is just above adjacent of 2 and 5

20 is divisible by 10.

[No need to connect (1, 10), (2, 20), (5, 10) ∵ transitive

Consider the Hasse diagram



$S = \{a, b, c, d, e, f, g, h\}$

$T = \{c, d, e\}$

lower bound of T :- $\{c, a, b\}$

g.l.b = c

upper bound of T :- $\{f, g, h\}$

l.u.b = f

Minimal elts are $\{a, b\}$

Maximal elts are $\{g, h\}$

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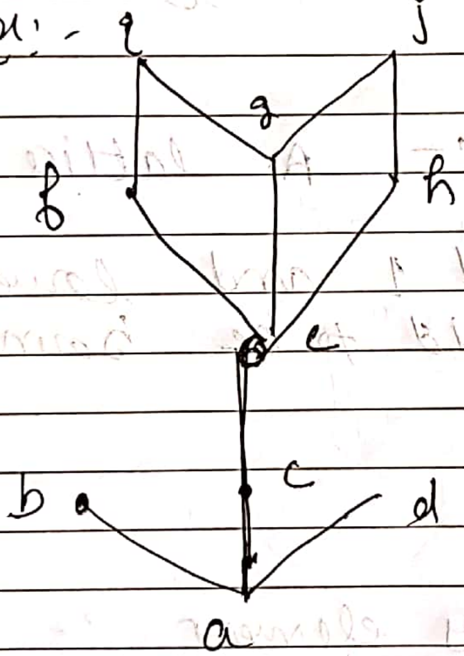
Lattice :-

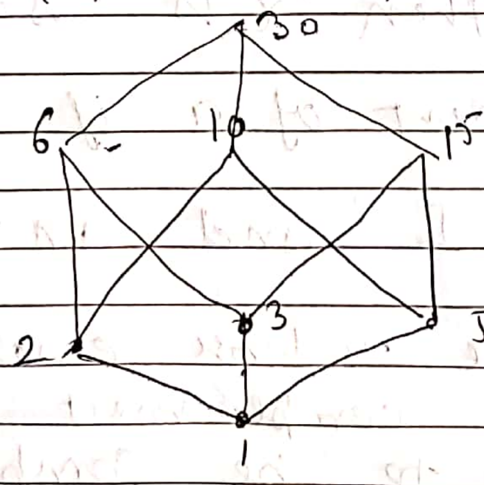
A partially ordered set $(L, \leq)$ is said to be lattice if every pair has g.l.b and l.u.b (Infimum) and (supremum)

Ex :-



Not a lattice as g.l.b of $\{b, i\} = a$ but l.u.b does not exist.

Ex $S = \{1, 2, 3, 5, 6, 10, 15, 30\}$ divisibility



It is a Lattice

Scanned with CamScanner

Note In lattice if $x, y \in A$ (where A is poset)

then
$x \vee y$ = Join of x and y = least upper bound

$x \wedge y$ = Meet of x and y = greatest lower bound.

Bounded Lattice:- A lattice with upper bound 1 and lower bound 0 is said to be bounded lattice.

Complement of any element:- Let L be a bounded lattice and $a \in L$ then x is said to Complement of a if:

$a \vee x = I$ and $a \wedge x = 0$

Note A lattice whose every element has a complement is said to be Complemented lattice.

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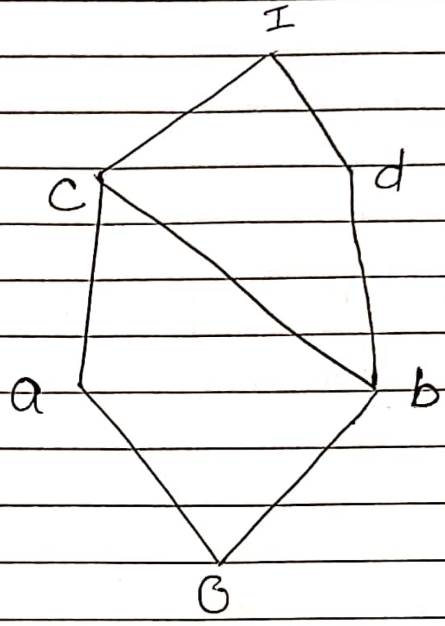
Ex:



Compliment of a is d

as $a \vee d = I$ & $a \wedge d = 0$

~~Suppose~~ Let's take c then
$a \vee c = c \neq I$ so c is rejected
for b. $a \vee b = c \neq I$ so b is also dejected

Complement of C :-

Since a is not complement of c ∵ $a \vee c = c$
b is " " " ∵ $b \vee c = c$

Let's check d.
$c \vee d = I$
but $c \wedge d = b \neq 0$

So Complement of c does not exist

Note Complement of $I = 0$ and
$0 = 1$